Non-selfadjoint spectral problems related to self-similar blowup in nonlinear wave equations

Kaori Nagatou
Karlsruhe Institute of Technology, Germany

We consider the wave equation with a power nonlinearity

$$u_{tt} - \Delta u = u^p,$$

with initial profiles $u(x,0)$ and $u_t(x,0)$, $x \in \mathbb{R}^3$, $t \geq 0$, and $p > 1$ an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form

$$u(x,t) = (T-t)^{-\frac{2}{p-1}} U\left(\frac{|x|}{T-t}\right), \quad T > 0,$$

with a smooth, radial profile U. In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the non-selfadjoint eigenvalue problem:

$$\mathcal{L}\mathbf{u} = \lambda \mathbf{u},$$

where $D(\mathcal{L}) \subset H^2_{\text{rad}}(B^3) \times H^1_{\text{rad}}(B^3)$, $H^k_{\text{rad}}(B^3) := \{\mathbf{u} \in H^k(B^3) : \mathbf{u} \text{ is radial}\}$,

$$\mathcal{L}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1'(\rho) - \alpha u_1(\rho) + u_2(\rho) \\ u_1''(\rho) + \frac{2}{\rho} u_1'(\rho) - \rho u_2'(\rho) - (\alpha+1) u_2(\rho) + V(\rho) u_1(\rho) \end{pmatrix},$$

$B^3 = \{x \in \mathbb{R}^3 : |x| \leq 1\}$, $\rho = \frac{|x|}{T-t}$, $V(\rho) = pU(\rho)^{p-1}$ and $\alpha = \frac{2}{p-1}$. We are interested in excluding eigenvalues of $\mathcal{L}$ in the crucial parts of the right complex half plane, which is ongoing work together with B. Schörkhuber, Y. Watanabe, M. Plum and M.T. Nakao. We first provide a compact set $R \subset \mathbb{C}$ such that no eigenvalues can exist in the right half-plane outside this set. Furthermore, we derive estimates ensuring that small discs with explicitly computable radii and carefully chosen centers are free of eigenvalues. Covering the set R with such discs gives, in principle, the desired non-existence of eigenvalues, implying (linear) stability.