グラフ上の逆問題、スペクトル解析と関連課題研究ワークショップ

- Date: June 16, 2018.
- Venue: School of Science, Bldg. 4, 501, Hokkaido University.

- **Mikhail Belishev (Steklov Institute, St. Petersburg), *Boundary control method in inverse problems on graphs: recent results and open problems.***

The *boundary control method* is an approach to inverse problems of mathematical physics based upon their close relations to the control and system theory. In the framework of the method, there are three approaches to problems on metric graphs.

The first approach makes the use of the relevant analogues of the *Gelfand-Levitan-Krein-Marchenko equations* on graphs. It enables one to recover a tree of inhomogeneous strings, the structure of the tree and variable densities of its edges being recovered from the boundary spectral and/or dynamical data [2].

The second approach is also workable for the trees and uses the same inverse data. It recovers the set of the so-called *zero wave diameter functionals*, which is bijective to the tree, and endows it with the proper metric, turning the set to an isometric copy of the graph under reconstruction. Also, this approach enables one to determine whether the graph contains cycles [1, 3].

The third approach exploits the relations with the C*-algebras. With the graph one associates the so-called *eikonal algebra*, which is determined by the hyperbolic dynamical system describing the wave propagation on the graph. We hope for fruitful connections between the topological invariants of the graph and algebraic invariants of its C*-eikonal algebra [4].

References

[1] M.I.Belishev. Boundary spectral inverse problem on a class of graphs (trees) by the BC-method. *Inverse Problems*, 20 (2004), No 3: 647–672.

[2] M.I.Belishev, A.F.Vakulenko. Inverse problems on graphs: recovering the tree of strings by the BC-method. *Journal of Inverse and Ill-Posed Problems*, 14 , no 1: 29–46, 2006.

[3] M.I.Belishev and N.Wada. On revealing graph cycles via boundary measurements. *Inverse Problems.*, 25 (2009), no 10, 105011, 1–25.

[4] M.I.Belishev and N.Wada. A C*-algebra associated with dynamics on a graph of strings. *J. Math. Soc. Japan*, Vol. 67, No. 3 (2015) pp. 1239-1274. doi: 10.2969/jmsj/06731239.

- **Leonid Pestov (Immanuel Kant Baltic Federal University), *Wave tomography problem with incomplete data.***

We consider inverse problem for wave equation: given hyperbolic Neumann-to-Dirichlet map with incomplete set of boundary sources to find variable sound speed $c(x)$ in a bounded domain Ω. We assume that sources and receivers are located at the same part of boundary $\partial\Omega$ and observation time is big enough. A linear reconstruction procedure of $c(x)$ based on the Boundary Control is proposed.

- **Akito Suzuki (Shinshu University), *Spectra of quantum walks on graphs.***

The Grover walk is one of the most studied quantum walks on graphs. In this talk, we define the walk and give a formula for the spectrum of the walk using a spectral mapping theorem from a random walk via the Joukowski transformation.

- **Kazunori Ando (Ehime University), *Discrete inverse problem for magnetic Schrödinger networks.***

We consider a boundary value problem of discrete magnetic Schrödinger operators on the 2-dimensional square lattice. We define the Dirichlet to Neumann map of this boundary value problem and show the reconstruction procedure for the potential and the 1-form on the boundary.

- **Hisashi Morioka (Doshisha University), *On embedded eigenvalues -discrete Laplacians and quantum walks-.***

In this talk, we discuss about recent works on eigenvalue problems for some discrete systems. First one is discrete Laplacians on some periodic lattices. In view of the scattering theory of Schrödinger operators with a small perturbation on $\mathbf{R}^d$, it is well-known that there is no positive eigenvalue. We consider similar problems for discrete Laplacians. For discrete systems, this problem is non-trivial. We derive the absence of eigenvalues embedded in the continuous spectrum for some lattices as well as some examples of embedded eigenvalues for some other kinds of lattices. We also derive a detection method of edge defects for position-dependent quantum walks using eigenvalues embedded in the continuous spectrum of time evolution operators.

- **Hiroaki Niikuni (Maebashi Institute of Technology), *Estimates of the number of eigenvalues embedded inside spectral bands of Schrödinger operators on carbon nanotubes with impurities.***

In this talk, we discuss the spectra of Schrödinger operators on carbon nanotubes with impurities from the point of view of the theory of quantum graphs. In the case of carbon nanotubes without impurities, it is known that the spectrum has the band-gap structure, namely, the spectrum consists of infinitely many closed interval (spectral bands) and the flat bands (the set of eigenvalues with infinite multiplicities). In this talk, we give a finite number of impurities expressed as the δ vertex conditions to the operator. As a result, we obtain additional eigenvalues embedded inside the spectral bands (not in the spectral gaps!).

Furthermore, we have an estimate from below of the number of embedded eigenvalues in each spectral bands for a suitable strength of δ vertex conditions.

In this talk, we consider the case where impurities are symmetric with respect to z-axis and rotation. Due to the rotational symmetry, we obtain a unitary equivalence between our operator and the direct sum of a finite number of Schrödinger operators on the degenerate carbon nanotube. Furthermore, we utilize the space-symmetry on z-axis and decompose those operators as the direct sum of the reducted operators on half size degenerate carbon nanotube with the Dirichlet and Newmann boundary condition. After those decomposition, we examine the estimate from below of the number of eigenvalues in the spectral gaps of each reduced operator. Finally, we show that those eigenvalues are embedded in the spectral bands of other reduced operators.

- **Yukihide Tadano (University of Tokyo),** ***Long-range scattering theory for discrete Schrödinger operators on several lattices.***

In this talk, we consider a scattering theory for discrete Schrödinger operators on several lattices, including square lattice, triangular lattice and the two-dimensional hexagonal lattice. In particular, the two-dimensional hexagonal lattice is one of the most interesting cases. Indeed, it is thought as a model of graphene, which is a sheet made of carbons arranged in the hexagonal lattice.

Let H_0, $H = H_0 + V$ be the corresponding discrete Schrödinger operator on a lattice without perturbation and perturbed with a potential V, respectively. Our interest is how to construct modified wave operators in the way of Isozaki and Kitada when V is long-range. First we introduce a result that, when a lattice is square or triangular lattice, the above Isozaki-Kitada modifiers exist and complete. Second we consider the two-dimensional hexagonal lattice, which cannot be treated in the first result. The reason comes from the number of atoms contained in the unit cell. The number also corresponds to the structure of Hilbert space equipped with the lattice. We construct Isozaki-Kitada modifiers and prove their existence and completeness. We also refer to difficulties which is specific to this case.

- **Igor Trushin (Tohoku University),** ***On inverse scattering on noncompact quantum graphs.***

I investigate inverse scattering problems on the graphs consisting of a compact part and a finite number of semi-lines attached. Graphs containing loops are of a prime interest.

- **Yuhsuke Yoshie (Tohoku University),** ***Controlling of graphs by periodicity of quantum walks.***

In this talk, we treat a kind of inverse problem from the viewpoint of quantum walks. Researches on localizations, linearly spreadings or search problem of quantum walks on fixed graphs have been suggested for recent years. On the contrary, our interest is to determine graphs inducing periodicity, an individual phenomenon of quantum walks, and find structure of such graphs. In particular, we will focus on the characterization of shape of graphs to induce odd-periodic quantum walks by the spectral method.